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MASV METHOD AND CONTROLING IN FINITE TIME

METODA MASP A ŘÍZENÍ V KONEČNÉM ČASE

Abstract

The paper is focused to problems with finite and infinite models and time optimization in the MASV method – Method of Aggregate State Variables.

Abstrakt

Článek je zaměřen na problémy nekonečných a konečných modelů a časové optimalizace v metodě MASP – Metoda Agregovaných Stavových Proměnných.

1 INTRODUCTION

The MASV method, called Method Aggregate State Variables, and its applications consist of four steps:

- mathematical model,
- control algorithm,
- simulation control,
- application in industry.

The paper contains some remarks to models and algorithms. Classical formulation of the MASV method does not solve the control in the finite time and construction algorithm uses the control in the infinite time. Time optimization cannot use this formulation. In the paper we show various ways how to solve these problems.

2 MATHEMATICAL MODEL OF CONTROL SYSTEM

The following mathematical model of the nominal nonlinear subsystem will be considered

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},t) + \boldsymbol{G}(\boldsymbol{x},t)\boldsymbol{u}, \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0, \tag{1}$$

where

$\boldsymbol{x} = [x_1, x_2, \ldots, x_n]^T$, $\dim \boldsymbol{x} = n$ is the vector function of state variables,

$\boldsymbol{u} = [u_1, u_2, \ldots, u_m]^T$, $\dim \boldsymbol{u} = m$ is the vector function of control variables,

$\boldsymbol{f} = [x_2, \ldots, f_{r_1}, x_{r_1+2}, \ldots, f_{r_2}, x_{r_2+2}, \ldots, f_n]^T$, $\dim \boldsymbol{f} = n$ is a continuous vector function,

$\boldsymbol{G}$, $\dim \boldsymbol{G} = (n, m)$ is the matrix of continuous functions $g_{i,j}(x)$,

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n – number of state variables (the order of the nonlinear subsystem),

n_j – partial order, m - the number of the control variables.

The matrix $\boldsymbol{G}$ is of the following form

$$\boldsymbol{G} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ g_{r_1 1} & g_{r_1 2} & \dots & g_{r_1 m} \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ g_{r_2 1} & g_{r_2 2} & \dots & g_{r_2 m} \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ g_{n1} & g_{n2} & \dots & g_{nm} \end{bmatrix} \qquad r_0 = 0; \quad r_j = r_{j-1} + n_j, \; j = 1,2,\dots,m, \quad n = r_m = \sum_{j=1}^{m} n_j$$

The condition of controllability of the nominal nonlinear subsystem (1) [Zítek & Víteček 1999]

$$\operatorname{rank} \boldsymbol{G}(\boldsymbol{x}, t) = m \tag{3}$$

is assumed.

It is supposed that $i = 1, 2,\dots, n$; $j = 1, 2,\dots, m$ and strictly not distinguished between a subsystem (system) and a model in the entire the following text.

3 CONTROL ALGORITHMS DESIGN – MASV METHOD

The task of the optimal tracking control design is determination of the feedback control

$$\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}, \boldsymbol{x}^w, t) \tag{5}$$

for the controllable nominal standard nonlinear subsystem (1), which for a given state trajectory $\mathbf{x}^w(t)$ ensures its tracking by a real state trajectory $\mathbf{x}(t)$ so that value of the quadratic objective functional

$$J = \int_0^\infty \left(\boldsymbol{e}^T \boldsymbol{D}^T \boldsymbol{D} \boldsymbol{e} + \dot{\boldsymbol{e}}^T \boldsymbol{D}^T \boldsymbol{T}^2 \boldsymbol{D} \dot{\boldsymbol{e}} \right) dt \tag{6}$$

$$\boldsymbol{e} = \boldsymbol{x}^w - \boldsymbol{x}, \; \dim \boldsymbol{e} = n, \qquad \lim_{t\to\infty} \boldsymbol{e}(t) = \lim_{t\to\infty} \dot{\boldsymbol{e}}(t) = 0 \tag{7}$$

is minimal, where $\boldsymbol{e}$ is the error vector,

$\boldsymbol{D}$ - the constant nonnegative aggregation matrix [dim $\boldsymbol{D}$ = (m,n), rank ($\boldsymbol{DG}$) = m],

$\boldsymbol{T}$ - the diagonal matrix of positive time constants T_j of the order m, i.e.

$$\boldsymbol{T} = \operatorname{diag}[T_1, T_2, \dots, T_m]. \tag{8}$$

By the method of the aggregation of the state variables, it is possible to obtain the optimal feedback control [Zítek & Víteček 1999]

$$\boldsymbol{u} = [\boldsymbol{DG}(\boldsymbol{x}, t)]^{-1} \{ \boldsymbol{T}^{-1} \boldsymbol{De} + \boldsymbol{D}[\dot{\boldsymbol{x}}^w - \boldsymbol{f}(\boldsymbol{x}, t)] \} \tag{9}$$

which causes the aggregated optimal closed-loop control system

$$\boldsymbol{D}\dot{\boldsymbol{e}} + \boldsymbol{T}^{-1} \boldsymbol{De} = 0, \quad \boldsymbol{e}(0) = \boldsymbol{e}_0 \tag{10}$$

and minimal value of the quadratic objective functional (6)

$$J^* = \boldsymbol{e}_0^T \boldsymbol{D}^T \boldsymbol{T}\boldsymbol{D}\boldsymbol{e}_0 \, . \tag{11}$$

If the elements d_{ji} of the aggregation matrix **D** will be chosen in accordance with the formulas

$$\left.\begin{array}{lll} d_{ji}=0 & \text{for} \quad i \le r_{j-1} & \text{or} \quad i > r_j \\ d_{ji}>0 & \text{for} \quad r_{j-1} < i \le r_j & \\ d_{jr_j}=1 & & \end{array}\right\}, \tag{12}$$

then the characteristic polynomial of the aggregated optimal closed-loop control system (10) can be written in the form

$$N(s) = \prod_{j=1}^{m} N_j(s) \, , \qquad N_j(s) = \left(\frac{1}{T_j} + s\right) \sum_{p=r_{j-1}+1}^{r_j} d_{jp} s^{p-r_{j-1}-1} \tag{13}$$

where

s is the complex variable in the Laplace transform,

N_j - the characteristic polynomial of the *j*-th autonomous control subsystem of the partial order n_j.

It is obvious that in this case the optimal closed-loop control system consists of m autonomous linear control subsystems whose desired dynamic behaviour can be ensured by a suitable choice of the time constants T_j and coefficients d_{ji} of their characteristic polynomials (13), i.e. by a suitable choice of the matrix $\boldsymbol{T}$ and $\boldsymbol{D}$. It is very important that the quadratic objective functional (6) has only an auxiliary purpose.

The feedback control (9) demands knowledge of the exact mathematical model of the nominal nonlinear dynamic subsystem (1). The control $\boldsymbol{u}$ is non-robust and is often called the equivalent control. It ensures the aggregated optimal closed-loop control system (10) from which after completion with equations

$$\dot{e}_i = e_{i+1}, \; i \neq r_j \tag{14}$$

the full optimal closed-loop control system can be obtained in the form with a matrix $\boldsymbol{A}$

$$\dot{\boldsymbol{e}} = \boldsymbol{A}\boldsymbol{e} \, , \tag{15}$$

which has the characteristic polynomial (13).

4 FORMULATION OF SOME OPEN QUESTION IN THE MASV METHOD

The MASV method was formulated considering

- infinite final time
- constant value of parameters $\boldsymbol{D}$, $\boldsymbol{T}$
- used method of energy optimization.

These properties are basic for development of the MASV method.

We will make some remarks to

- finite and infinite models,
- control in the finite time,
- control with error in the finite time,
- optimal control in the time.

5 MODELS WITH INFINITE AND FINITE FINAL TIME AND WITH ERROR IN FINAL TIME

We will rate models according the final time and the error in final time.

Model MASV(∞, 0) – the classical MASV method.

$$J = \int_0^{\infty} \left(\boldsymbol{e}^T \boldsymbol{D}^T \boldsymbol{D}\boldsymbol{e} + \dot{\boldsymbol{e}}^T \boldsymbol{D}^T \boldsymbol{T}^2 \boldsymbol{D}\dot{\boldsymbol{e}} \right) dt$$

(16)

$$\lim_{t\to\infty} \boldsymbol{e}(t) = \lim_{t\to\infty} \dot{\boldsymbol{e}}(t) = 0 \qquad (17)$$

The final time is infinite and the error in infinity equals to **0.**

The infinite final time is not realistic.

Model MASV (t_f,0) – basic version with infinite final time and zero error

$$J = \int_0^{t_f} \left(\mathbf{e}^T \mathbf{D}^T \mathbf{D}\mathbf{e} + \dot{\mathbf{e}}^T \mathbf{D}^T \mathbf{T}^2 \mathbf{D}\dot{\mathbf{e}} \right) dt \qquad (18)$$

$$\mathbf{e}(t_f) = \dot{\mathbf{e}}(t_f) = 0 \qquad (19)$$

This model is realistic, but is not usually solvable.

Model MASV (t_f**, error)** **-** version with the finite final time t_f and limited value of $\mathbf{e}(t_f), \dot{\mathbf{e}}(t_f)$

$$J = \int_0^{t_f} \left(\mathbf{e}^T \mathbf{D}^T \mathbf{D}\mathbf{e} + \dot{\mathbf{e}}^T \mathbf{D}^T \mathbf{T}^2 \mathbf{D}\dot{\mathbf{e}} \right) dt \qquad (20)$$

$$\left\| \boldsymbol{e}(t_f) \right\|_0 \le error_o, \left\| \dot{\boldsymbol{e}}(t_f) \right\|_1 \le error_1 \qquad (21)$$

In this model the final time t_f is finite and the error **e** is less that constants **error.**

6 DESIGN CONTROL ALGORITHM MASV(t_f, error)

The aim is to find t_f such that the error **e** in the final time is less than **error**. One type of solution is the following algorithm. The main idea is to transform the problem by the method MASV.

Control algorithm :

- find control by using method MASP,
- compile the system of differential equations for errors,
- compute a priori estimate for the solution,
- from a priori estimate calculate the value of t_f

7 TIME eps - OPTIMIZATION USING THE METHOD MASV(t_f , error)

Formulation of the problem - Find the minimal time t_f , which meets the MASV Model (t_f**, eps**).

Remarks to solution of the problem.

Design control algorithm MASV (t_f, **error)**, choose a priori estimate for solution of this problem.

One of the possible ways how to solve this problem is optimizing variables using a priori estimate. We find approximation of solution.

8 CONCLUSION

The paper describes using the MASV method in the models with non-standard condition and algorithms, where the final time is finite and the optimization is in the time.

These ideas will be developed and employed on realistic problems in the next author's paper.

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